# Intermediate Algebra Review of Key Ideas

## *Exponents and the Order of Operations*

**Exponential notation:** Exponents are shorthand for repeated multiplication.

**Example 1**: $3 \cdot 3 \cdot 3 \cdot 3 = 3^4$ 4 is the **exponent** and 3 is the **base**. Note: Exponents apply only to the symbol directly below.

**Example 2**:

a) $-3^2 =$

b) $(-2)^2 =$

**Example 3**: Evaluate each expression.

a) $-5^2$

b) $(-5)^2$

c) $-(-5)^2$

d) $-\left(-5^2\right)$

**Order of Operations:**

1. Perform all calculations within grouping symbols
2. Simplify all exponential expressions (and roots)
3. Perform all multiplication and division, in order, from left to right.
4. Perform all addition and subtraction, in order from left to right.

| **Grouping symbols** | |
|---|---|
| **Exponents** | **Roots** |
| **Multiplication** | **Division** |
| **Addition** | **Subtraction** |

**First** ↓ **Last**

Inverse operations

**Example 1**: Evaluate each expression.

a) $\left[(4-2)^2+2\right]^2 \cdot 4-3$

b) $3^3 \div 3^2 - 4(1-6) \div 5$

## *Evaluating Variable Expressions*

**Definitions: A variable** is a letter that can represent different numbers. A **constant** is a number that stays the same. **An algebraic expression** consists of variables and/or numerals, often combined with operation signs and grouping symbols.

**Example 1**: $3ab(c+2)$

**Definition:** The **value** of an expression is the number represented by the expression. **Evaluating** an expression means finding that value or number.

**Example 2**: Evaluate the following expression for the given value of the variable.
$3x-5+4x$ for $x=-1$

**Example 3**: Evaluate the following expression for the given value of the variable.
$3x^2-6x-4$ for $x=-2$

**Example 4**: Evaluate the following expression for the given value of the variable.

$3(x+y)^2+2(x+y)-4$ for $x=2$ and $y=4$

**KEY WORDS used for translating operations**

| ADDITION | SUBTRACTION | MULTIPLICATION | DIVISION |
|---|---|---|---|
| add | subtract | multiply | divide |
| sum | difference | product | quotient |
| plus | minus | times | divided by |
| more than | less than | twice | ratio of |
| increased by | decreased by | of | per |

## *Sets of Numbers*

**Natural Numbers:** These are the counting numbers: **(1, 2, 3,...)**

**Prime Numbers:** These are numbers that have *exactly* two distinct factors, the number itself and 1.

**Example 1**: The prime numbers are {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, …}

**Whole numbers:** This set includes the natural numbers and 0: **{0, 1, 2, 3....}**

**Composite:** A natural number other than 1 that is not a prime number. Every composite number can be factored into a product of a prime numbers. This is called the **prime factorization** of that composite number. All natural numbers are either prime or composite.

**Example 2**: The prime factorization of 36 is $36 = 4 \cdot 9 = 2 \cdot 2 \cdot 3 \cdot 3 = 2^2 \cdot 3^2$

**Integers:** This set contains all the whole numbers and their opposites: **(...-1,-2-3 , 0, 1,2,3...)**

**Rational Numbers:** Numbers of the form $\frac{p}{q}$ where *p* and *q* are integers. This set contains fractions and terminating and repeating decimals in addition to the integers.

**Terminating decimals:** These occur when the division implied by the fraction ends with the remainder of 0.

**Example 3**: The number $\frac{44}{8} = 5.5$ is a terminating decimal

**Repeating decimals:** These occur when a remainder reappears.

**Example 4:** The number $\frac{1}{3} = 0.333\overline{3}$ is a terminating decimal

**Irrational numbers:** These are the real numbers whose decimal expansions don't terminate or repeat. For example, $\sqrt{2}$ and $\pi$ .

**Real numbers:** The rational and irrational numbers together make up the real numbers. These correspond to all the points on a number line.

## *Rules for Operations on Real Numbers*

**Definition: Absolute value** can be thought of as the distance a number is from 0 on a number line. **Distance is never negative.**

**Example 1**: Evaluate the following expressions.

a) $|-8|$

b) $-|8|$

**Definition: Inequalities** are any expressions dealing with greater than or less than signs.

**Example 2**: The statements $2 < 5$, $3 > -5$, $x \geq 5$, and $x \leq -5$ are all inequalities.

**Adding Positive Numbers:** add as usual, answer is positive. **(+ and + = +)**

**Example 3**: $3.24 + 2.78 =$

**Adding Negative Numbers:** Add the absolute values of the numbers and make the answer negative. **(- and - = -)**

**Example 4**: $(-5.23) + (-5) =$

**Adding Positive and Negative numbers:** Subtract the absolute value of the smaller number from the absolute value of the larger number. The sign of the result is the same as the original sign of the number with the larger absolute value.

**Example 5**: $(-10) + (6) = -(|10| - |6|) = -4$

↖ Negative since $|-10| > |6|$

**Example 6:** Adding Positive and Negative numbers

a) $(12) + (-8)$

b) $-12 - (-8)$

**Opposites (or additive inverses):** Whenever opposites are added the result is 0 and whenever two numbers add to 0, those numbers are opposites.

**Example 7:** $-6+6=0$ So 6 and –6 are opposites.

**Additive Inverse Property (Law of Opposites):** When opposites (or additive inverses) are added, their sum is 0. $a+(-a)=0$

**Rules for Multiplying Signed Numbers:**

**Positive** times **Negative** = **Negative**
**Negative** times **Negative** = **Positive**
**Positive** times **Positive** = **Positive**

**Terminology of Fractions**

**Identity Property of 1:** Any number multiplied by 1 equals the same number, $1 \cdot a = a$.

**Simplifying Fractions:** Factor out any common factors from both the numerator and denominator, and then use the identity property of 1. The fraction is "reduced" if there are no more common factors.

**Adding and Subtracting Fractions with Like Denominators:** When the denominators are the same, numerators are added or subtracted and placed over the common denominator.

**Adding and Subtracting Fractions with Unlike Denominators:** When the denominators are not the same, we first have to find a common denominator, convert the fractions to equivalent fractions with the LCD as the new denominator and then add as before.

**Definition:** The **reciprocal** or **multiplicative inverse of a number** $a$ $(a \neq 0)$ is the number $\frac{1}{a}$. The important idea is that the two numbers multiply to 1. The reciprocal of a fraction such as $\frac{a}{b}$ is the fraction $\frac{b}{a}$, since $\frac{a}{b} \cdot \frac{b}{a} = 1$.

**Dividing fractions:** Multiply the first fraction by the reciprocal of the second fraction. We usually say " invert and multiply." Be sure to simplify if possible.

**Multiplication of Fractions:** To multiply two fractions, we multiply the numerators and multiply the denominators: $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$. Be sure to simplify.

**Example 1**: **Perform the indicated operation**

a) $\frac{7}{5} \cdot \frac{15}{14}$

b) $\left(\frac{9}{20}\right)\left(\frac{5}{-3}\right)$

c) $\frac{2}{4} + \frac{5}{4}$

d) $\frac{5}{4} - \frac{2}{4}$

e) $\frac{2}{4} + \frac{5}{3}$

f) $\frac{5}{12} - \frac{1}{8}$

g) $\frac{15}{4} \div \frac{5}{12}$

h) $\frac{18}{7} \div \frac{-3}{14}$

**Division and Zero**

**Fact**: 0 divided by any non-zero number $a$ is 0: $\frac{0}{a} = 0$

**Fact**: *Division by zero is not allowed.*

**Division by Zero**: Suppose $a$ is any non-zero number. Then:

a) $\frac{a}{0}$ is undefined. Suppose you could divide, then there would be a number $b$ for which $\frac{a}{0} = b$. We could then rewrite the division as the related multiplication $a = b \cdot 0$. But this would mean that *a* would have to be zero! So there is no number *b* that will make this statement true.

b) $\frac{0}{0}$ is indeterminate. Suppose you could divide, then there would be a number $b$ for which $\frac{0}{0} = b$. We could then rewrite the division as the related multiplication $0 = b \cdot 0$. But this is true for *any* number *b*.

## The Properties of Real Numbers

### The Commutative Properties

Changing the order in addition and multiplication does not change the result. For example: $3+5=5+3=8$ and $3\cdot 5=5\cdot 3=15$

**Commutative Property of Addition** (CPA): For any real numbers *a* and *b*,

$$a+b=b+a\,.$$

**Commutative Property of Multiplication** (CPM): For any real numbers *a* and *b*,

$$a\cdot b=b\cdot a\,.$$

### The Associative Properties

Changing the grouping in addition and multiplication does not change the result. For example: $3+(5+7)=(3+5)+7$ and $2\cdot(3\cdot 5)=(2\cdot 3)\cdot 5$

**Associative Property of Addition** (APA): For any real numbers *a*, *b*, and *c*:

$$(a+b)+c=a+(b+c)\,.$$

**Associative Property of Multiplication** (APM): For any real numbers *a*, *b*, and *c*:

$$(a\cdot b)\cdot c=a\cdot(b\cdot c)\,.$$

### The Distributive Property

The distributive property provides the link between multiplication and addition.

**The Distributive Property:** For any real numbers *a*, *b*, and *c*:

$$a(b+c)=a\cdot b+a\cdot c$$

product of factors    sum of terms

If we perform the multiplication $a(b+c)=a\cdot b+a\cdot c$, then we are **distributing.** If we start with the sum and get a product $a\cdot b+a\cdot c=a(b+c)$, then we are **factoring out** the common factor $a$.

## Solving Equations

**Definitions**: An **equation** is a statement about the relationship between two algebraic expressions. The truth of an equation may depend upon the value of the variable or variables. **Solving** an equation means finding the value(s) of the variable that make the equation true.

**Example 1:** $x = 2$ is a solution of the equation $x + 5 = 7$ because, when 2 is substituted in for *x* the result is a true statement: $2 + 5 = 7$ is true.

**Definition**: **Equivalent equations** are equations that have the same solution.

We will use the properties of equality to produce equations that are equivalent to the given equation in order to find the solution.

**The Addition Property of Equality**: $a = b$ is equivalent to $a + c = b + c$. This says that you can add the same number to both sides of an equation without changing the solution.
**The Multiplication Property of Equality**: $a = b$ is equivalent to $ac = bc$ if $c \neq 0$. This says that you can multiply both sides of an equation by the same non-zero number without changing the solution.

**Example 2**: Solving Equations with the Addition Property

Solve: $x - 5.3 = 7.27$.

**Example 3**: Solving Equations with the Multiplication Property
Solve: $-\frac{5}{3}x = \frac{7}{4}$.

**Example 4**: Solving Equations Using Both Properties

Solve: $7 - \frac{1}{2}x = 18$.

**Combining Like Terms**

The parts of an algebraic expression that are being added or subtracted are called **terms**. **Like terms** are either constant terms or terms containing the same variable raised to the same power. We use the distributive property in order to combine like terms. We factor out the common factor and add the numerical coefficients.

$$5.1x^2 + 3.3x^2 + 5x + 3x = (5.1 + 3.3)x^2 + (5 + 3)x = 8.4x^2 + 8x$$

**Example 5:** Combining Like Terms

**Simplify:** $-2x + 3[5 + 2(-3x + 2y)]$

**Example 6:** Combining Like Terms

**Simplify:** $-(3y - 2x) - 2(-2x + y)$

# Solving Equations Containing Grouping Symbols

To solve a linear equation in one variable:

1. Simplify to terms, that is, remove grouping symbols.
2. Combine like terms on each side of the equation
   Note: You may need to repeat the first two steps several times to remove *all* the grouping symbols.
3. Move any terms that contain the variable to one side by adding inverses to both sides of the equation. (Addition Property of Equality)
4. Isolate the variable:
   a. First, move all constants being added to or subtracted from the variable term to the other side. (Addition Property of Equality)
   b. Finally, complete the solution by multiplying both sides of the equation by the reciprocal of the coefficient of the variable. (Multiplication Property of Equality)
5. Check the solution – Replace the variable with the value you found and simplify both sides of the equation. If the resulting number on each side of the equation is the same, then the solution is correct.

**Example 7**: Solving Equations Containing Grouping Symbols.

Solve: $\frac{2}{3}(x-2)-1=\frac{1}{4}(x-3)$

**Example 8**: **Solving Linear Equations in One Variable**

Solve: $3[2-4(x-1)]=3-4(x+2)$