Rules of Exponents

If x, y are real numbers and m, n are positive integers then:

Product Rule	$x^m \cdot x^n = x^{m+n}$	$x^2 \cdot x^3 = x^{2+3} = x^5$
Quotient Rule	$\frac{x^m}{x^n} = x^{m-n}$	$\frac{x^5}{x^3} = x^{5-3} = x^2$
Power Rule	$\left(x^m\right)^n = x^{m \cdot n}$	$\left(x^2\right)^3 = x^{2 \cdot 3} = x^6$
Zero Exponent	$x^0 = 1, x \neq 0$	$\left(x^2\right)^0 = 1$
Distributive over Multiplication	$(xy)^n = x^n y^n$	$(-2x)^5 = (-2)^5 x^5 = -32x^5$
Distributive over Division	$\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$	$\left(\frac{2x}{-3y}\right)^3 = \frac{(2x)^3}{(-3y)^3} = \frac{8x^3}{-27y^3}$
Negative Exponent	$x^{-n} = \frac{1}{x^n}, x \neq 0$	$x^{-11} = \frac{1}{x^{11}}$

Example 1: Multiplication and the rules of exponents

Simplify each expression.

a) $3^3 \cdot 3^2$

b) $x^4 \cdot x^2$

c) $\left(2x^4y^3\right) \cdot \left(-3xy^2\right)$

Example 2: Division and the rules of exponents

a) $\frac{2^5}{2^2}$

b) $\frac{x^4}{x^2}$

c) $\frac{20x^6}{-4x^2}$

d) $\frac{-10x^5y^4}{4x^4y^2}$

Example 3: Negative Exponents

a) $(-3)^{-2}$

b) $(-2)^{-3}$

c) $7x^{-4}$

d) $2^{-3} \cdot 2^{2}$

e) $\left(x^{-4}y^{3}\right) \cdot \left(x^{2}y^{-5}\right)$

f) $\dfrac{x^{4}}{x^{-2}}$

e) $\dfrac{x^{-4}}{x^{-2}}$

f) $\dfrac{x^{-2}}{x^{-4}}$

g) $\dfrac{5x^{-8}}{y^{-4}z^{2}}$

Example 4: Powers and the distributive rule of exponents

a) $\left(2^{4}\right)^{5}$

b) $\left(x^{4}\right)^{3}$

c) $\left(-y^{2}\right)^{4}$

d) $\left(a^{2}b^{5}\right)^{2}$

e) $\left(-2x^{-3}y^{4}\right)^{2}$

f) $\left(2ab^{-2}\right)^{5}\left(-2a^{3}b^{4}\right)^{3}$

Example 5: Second distributive rule of exponent

a) $\left(\dfrac{2}{y}\right)^{3}$

b) $\left(\dfrac{2^{3}}{3^{2}}\right)^{2}$

c) $\left(\frac{-y^2}{x^5}\right)^4$

d) $\left(\frac{ab^4}{c^3}\right)^3$

e) $\left(\frac{-2x^2y}{3z^3}\right)^3$

f) $\left(\frac{-2x^2y}{3z^3}\right)^0$

g) $\left(\frac{-2x^{-2}y}{3z^3}\right)^3\left(\frac{-x^2z^2}{2y^{-4}}\right)^{-2}$

h) $\left(\frac{5x^2y}{az^2}\right)^{-2}$