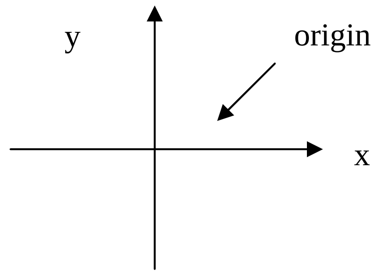
Section 3 Graphs and the Cartesian Coordinate System

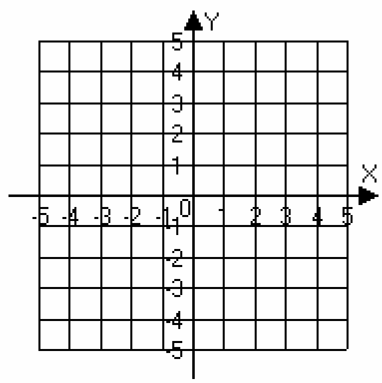
To visualize data with two variables we use the **Cartesian plane**. This consists of two perpendicular axes (*x*-axis and the *y*-axis). The point where the two axes cross is called the **origin**. An **ordered pair** (x, y) is used to list the two coordinates of a point. The point (3,4) has an *x*-coordinate of 3 and a *y*-coordinate of 4.



The axes divide the plane into four **quadrants**.

Example 1: **Plot the following points and identify the quadrant in which they lie**

a) $(-2, 3)$ b) $(-4, -2)$ c) $(3, -2)$



Definitions: A **solution** of an equation in two variables is an ordered pair that makes the equation true.

Example 2: **Determine if the ordered pair is a solution of the equation**

a) $(-2, -3)$
$y = 3x + 3$

b) $(-4, -2)$
$y = 3x + 3$

c) $(-2, 7)$
$y = x^2 + 3$

Definitions: The **solution set** for an equation in two variables is the set of all ordered pairs for which the equation is true. The **graph** of the equation is a picture of the set of all ordered pairs that solve the equation.

Graphing by Plotting Points

One way to draw the graph of an equation is to make a table of values containing some of the ordered pairs in the solution set and then drawing the graph.

Example 3: **Graphing by plotting points**

Sketch a graph of the following equations by making a table of values and plotting the points.

a) $y = x$

b) $y = -x$

c) $y = 2x$

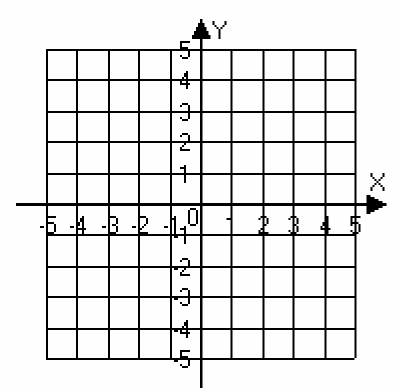
d) $y = -\frac{1}{2}x$

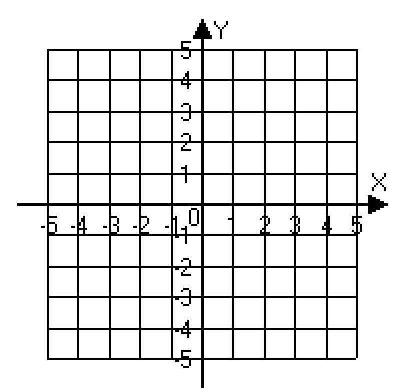
e) $y = 3x - 2$

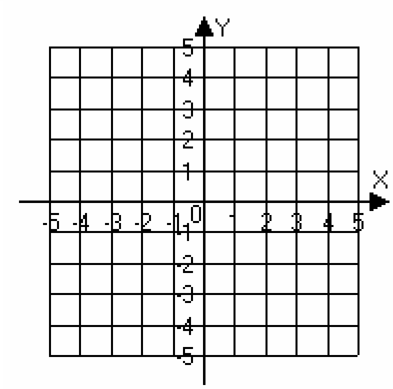
f) $y = |x|$

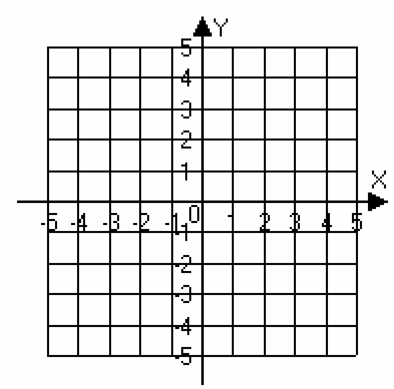
g) $y = x^2 - 4$

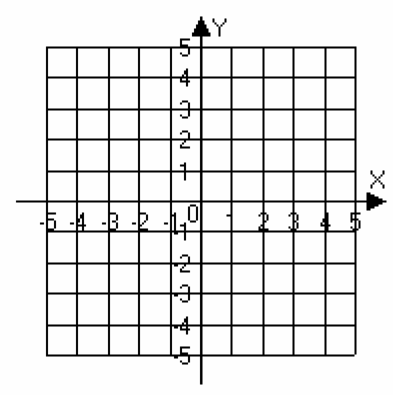
h) $y = x^2 + 2$







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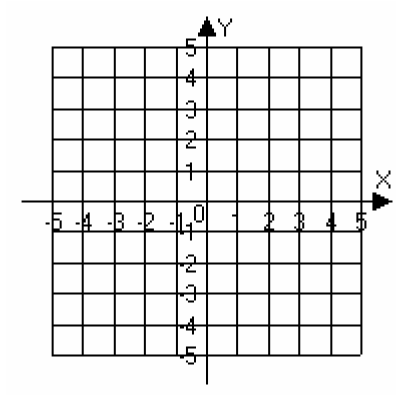
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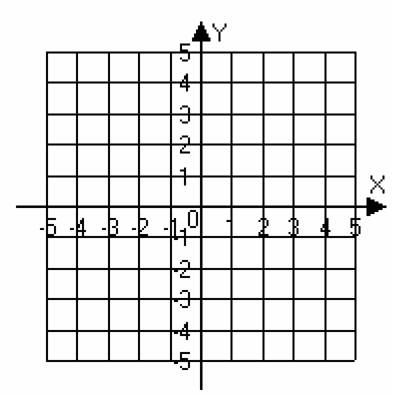
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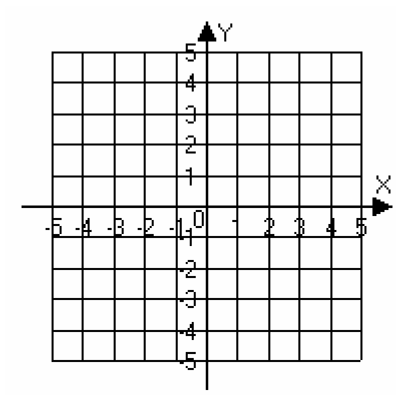
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Definitions: The ***x*-intercept** of a line is the point where its graph meets the *x*-axis. To find the *x*-intercept, set $y = 0$ and solve for *x*. The ***y*-intercept** of a line is the point where its graph meets the *y*-axis. To find the *y*-intercept, set $x = 0$ and solve for y.

Example 4: Finding *x*- and *y*-intercepts

a) Find the *x*- and *y*-intercepts of $3x - 5y = 11$.

b) Find the *x*- and *y*-intercepts of $-2x + 12y = 12$.

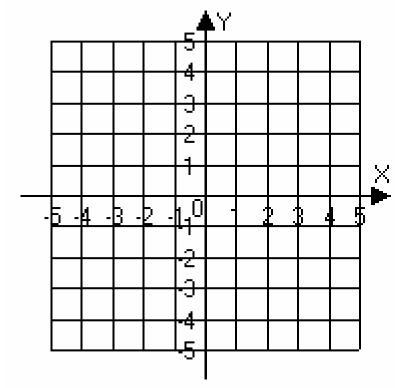
c) Find the *x*- and *y*-intercepts of $y = -2x + 12$.

We will use two basic techniques of graphing:

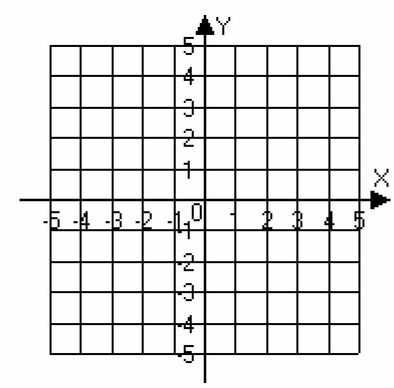
1. Find the intercepts and graph the line.
2. 2. Make a table of values by substituting in several values of *x*, plot the points, and then draw the line.

Example 5: Graphing by using intercepts.

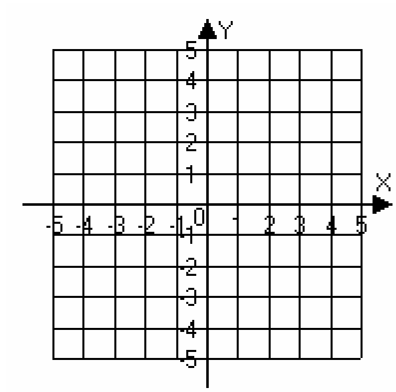
For each of the following, find the *x*-intercept and *y*-intercept and graph.



a) $2x - 4y = 6$ a)



b) $-3x - 2y = 6$ b)



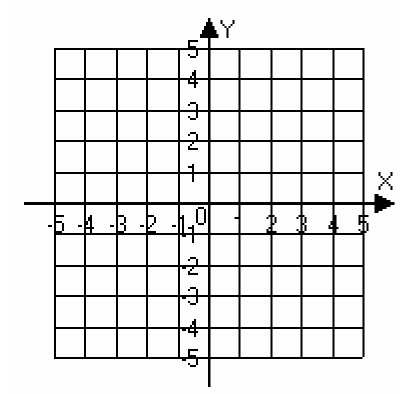
c) $3x + 4y = 12$ c)

<u>Special Cases</u>: Horizontal and Vertical Lines

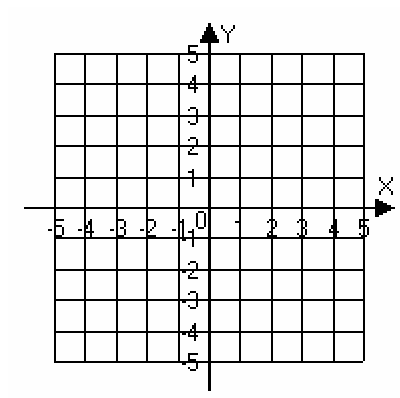
1. The graph of $x = a$ is a **vertical line** that passes through $(a, 0)$.
2. The graph of $y = b$ is a **horizontal line** that passes through $(0, b)$.

Example 6: **Graphing horizontal and vertical lines**

Graph the following lines.



a) $x = 3$ a)



b) $y = -2$ b)

Rates

Definition: A **rate** is a ratio that compares two quantities with different units.

Example 1: Writing a rate.

If you earn \$55 for 11 hours of work, then the ratio of dollars earned to hours worked is $\frac{55 \text{ dollars}}{11 \text{ hours}}$.

Definition: The ratio of some quantity to 1 is called a **unit rate**.

Example 2: Writing a unit rate.

If you earn \$55 for 11 hours of work, then the ratio of dollars earned to hours worked is $\frac{55 \text{ dollars}}{11 \text{ hours}} = \frac{5 \text{ dollars}}{1 \text{ hour}}$. This is a unit rate.

Example 3: Writing unit rates

Write as units rates.

a) 205 calories for 7 grams of fat.

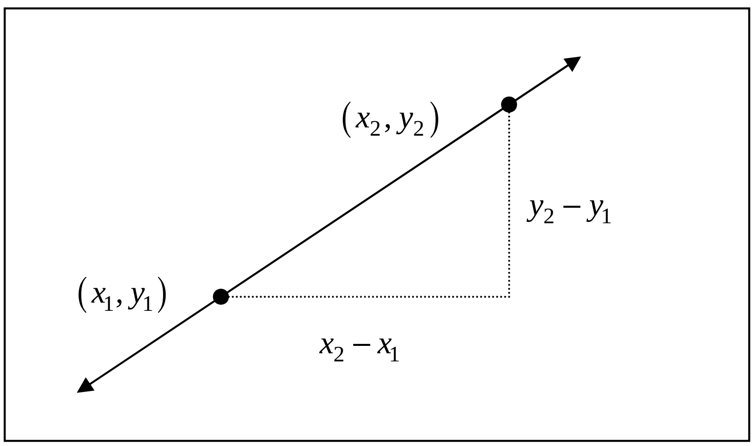
b) 410 miles in 7 hours

c) A pulley makes 63 complete rotations in 18 seconds. How many rotations per second does the pulley make?

Slope

Definition: The **slope** of a line through two points is the ratio of the vertical change to the horizontal change. The slope of the line through the points (x_1, y_1) and (x_2, y_2) is

slope = $m = \dfrac{\textit{vertical change}}{\textit{horizontal change}} = \dfrac{\Delta y}{\Delta x} = \dfrac{y_2 - y_1}{x_2 - x_1}$



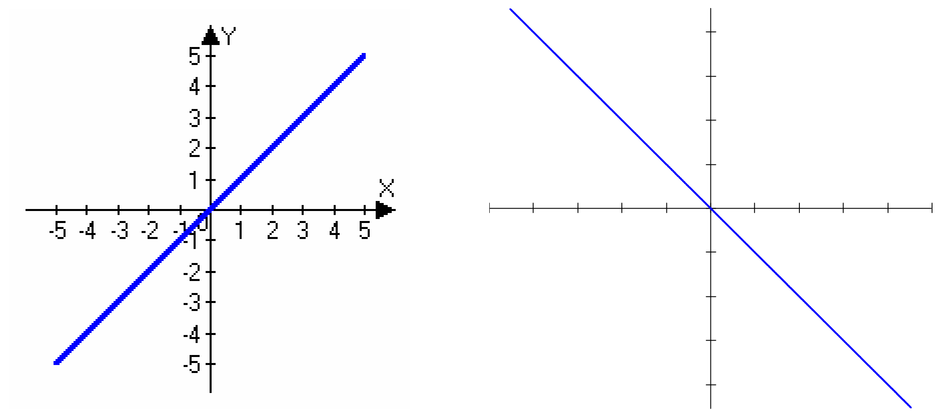
Example 1:

a) Find the **<u>slope</u>** of the line joining the points $(5, -3)$ and $(2, 3)$.

b) Find the **<u>slope</u>** of the line joining the points $(-2, 3)$ and $(-5, 5)$.

Facts about Slope:

1. The slope of a horizontal line is zero.
2. A line that rises going from left to right has **positive slope**.
3. A line that falls going from left to right has **negative slope**.
4. The slope of a vertical line does not exist. Why?

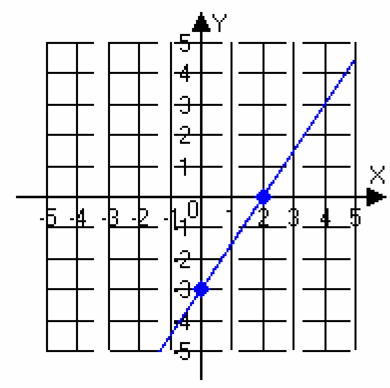


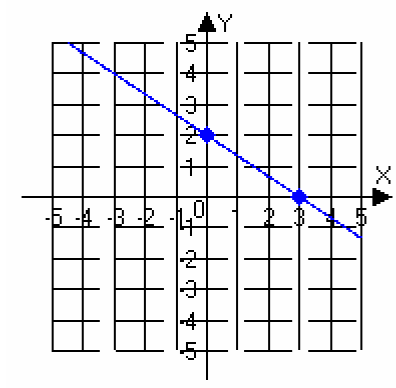
positive slope   negative slope

Example 2: Find the slope of the line joining the points $(2, 3)$ and $(-5, 3)$

Example 3: Find the **slope** of the line joining the points $(3, -3)$ and $(3, -2)$.

Example 4: Finding the slope of a line given the graph.





Slope-Intercept Form of the Equation of a Line

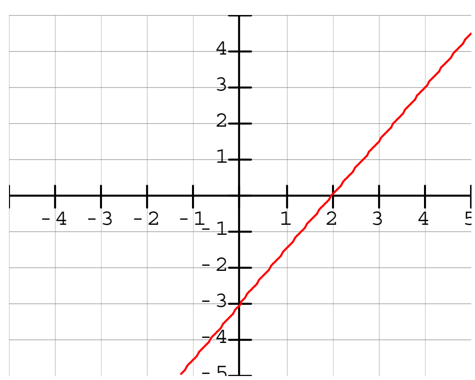
Definition: The **slope-intercept form** of the equation of a line is $y = mx + b$, where m is the slope and b is the *y*-intercept.

Example 1: Slope-intercept form.

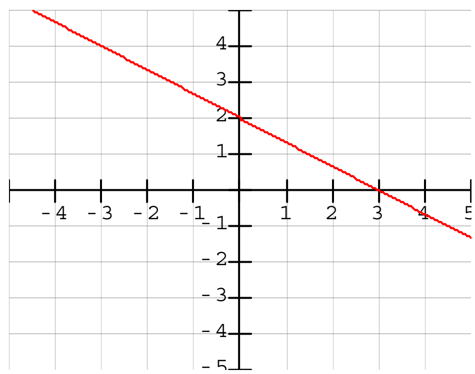
a) Determine the slope and *y*-intercept of $3y - 2x = 7$.

b) Give the slope-intercept equation for the line that passes through the point $(0, 4)$ with slope $m = 2/5$.

c) Find the slope-intercept equation for the line whose graph is given.



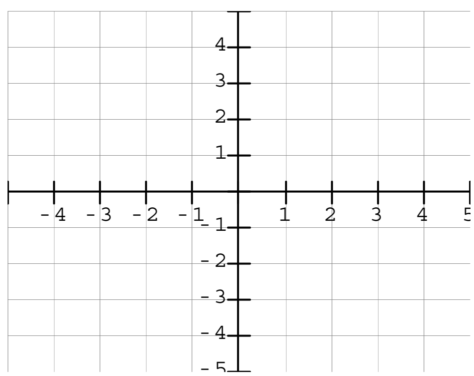
d) Find the slope-intercept equation for the line whose graph is given.



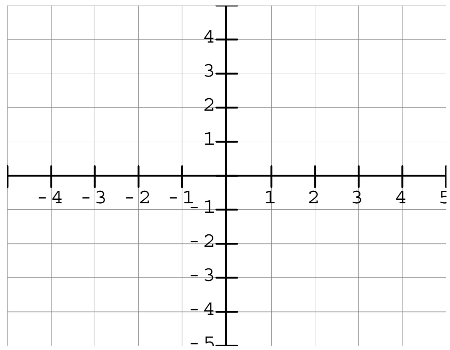
Example 2: Graphing using slope and y-intercept.

Graph the line from the given information.

a) Graph the line with slope $2/5$ and y-intercept $(0, -2)$.



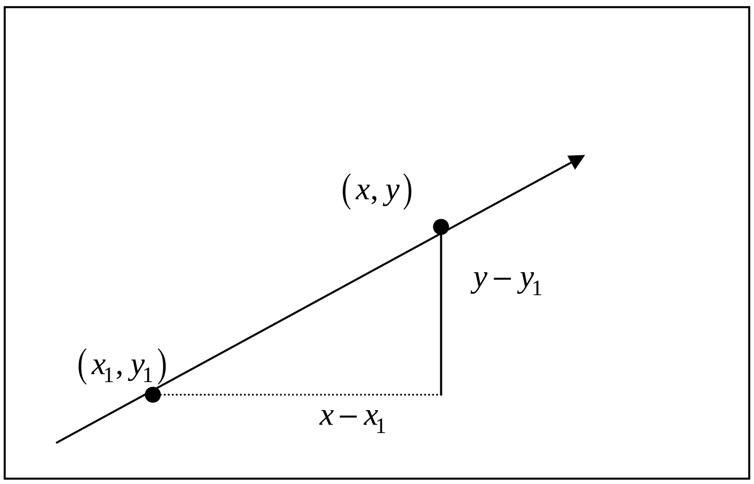
b) Determine the slope and *y*-intercept of $4y - 2x = 8$ and then graph.



Important Facts:

1. Two lines are **parallel** lines if they have the same slope.
2. Two lines are **perpendicular** lines if the product of the slopes is -1. That is $m_1 \cdot m_2 = -1$

Point-Slope Form of the Equation of a Line



The point-slope form is found by applying the slope formula to the graph above:

$$m = \frac{y - y_1}{x - x_1}$$

If we put the *y*'s on one side:

$$y - y_1 = m(x - x_1)$$

Exercise:

a) Write an equation for the line that passes through the points (-2,5) and (4, -2).

b) Write an equation for the line that passes through the points (-2,5) and (-2, -2).

c) Write an equation for the line that passes through the points (-2,5) and (4,5).

d) Write an equation of the line parallel to the line $-3x+5y=11$ and passing through $(-2,3)$.

e) Write an equation of the line perpendicular to the line $-3x+5y=11$ and passing through $(-2,3)$.