

## Section 4 Functions

**Definition:** A **function** is a collection of ordered pairs in which no two ordered pairs have the same first coordinate and different second coordinates. We can also say that a function is a correspondence between two sets such that each element in the first set is paired with exactly one element in the second set.

**Definition:** The **domain** of a function is the set of all possible values of the first coordinate (independent variable).

**Definition:** The **range** of a function is the set of all possible values for the second coordinate (dependent variable). If  $x$  is the input value and the function  $f$  is given, then **the value of  $f$  at  $x$**  (or  $f$  of  $x$ ) is  $f(x)$ .

### Example 1 Functions

Determine if each set is a function.

a)  $f = \{(0,1), (1,2), (2,3)\}$

b)  $g = \{(0,1), (0,2), (2,3)\}$

The domain of the function  $f$  defined in a) is  $\{0,1,2\}$

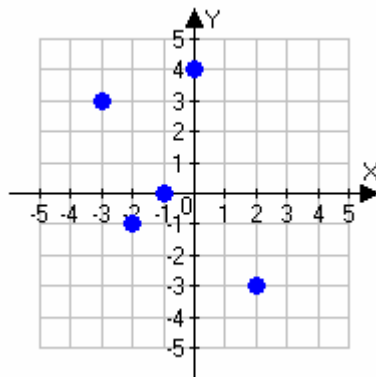
**Definition** The **graph** of a function  $f$  with domain  $A$  is a set of ordered pairs.

**Theorem: Vertical Line Test:** If a vertical line cannot be drawn so that it touches a graph in more than one point, then the graph is the graph of a function.

### Example 2 Graphs and Functions

For the graph below:

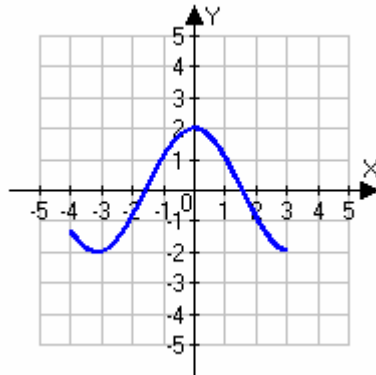
- Use the vertical line test to determine if the graph is a function.
- Find the domain
- Find the range



### Example 3 Graphs and Functions

For the graph below:

- d) Use the vertical line test to determine if the graph is a function.
- e) Find the domain
- f) Find the range



### Function Notation

$f(x)$  denotes the value of the function  $f$  at  $x$ . It is a number in the range of the function  $f$ . If  $y = f(x)$  then we say that  $y$  is  $f$  of  $x$ . To evaluate a function, substitute the appropriate value of  $x$  into the formula.

### Example 4 Evaluating Functions

Let  $f(x) = 2x + 1$  and  $g(x) = -3x^2 + x$ . Find the following:

- a)  $f(-2)$
- b)  $g(-2)$
- c)  $f(0) + g(0)$
- d)  $f(t)$
- e)  $g(a + h)$

**Example 5 Finding Domains of Functions**

Find the domain of each function.

a)  $f(x) = |x|$

b)  $g(x) = \frac{x + 2}{x - 5}$

c)  $f(x) = x^3$

d)  $f(x) = \sqrt{x + 1}$