Section 5 Lines

Definitions: The ***x*-intercept** of a line is the point where its graph meets the *x*-axis. To find the *x*-intercept, set $y = 0$ and solve for *x*. The ***y*-intercept** of a line is the point where its graph meets the *y*-axis. To find the *y*-intercept, set $x = 0$ and solve for y.

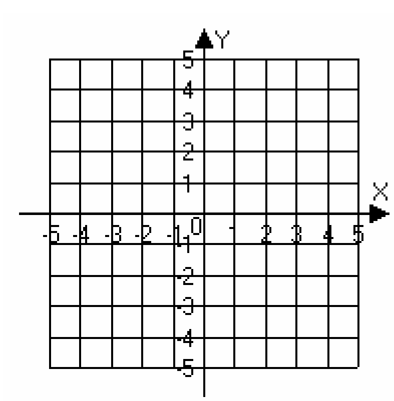
Example 1: Finding *x*- and *y*-intercepts
Find the *x*- and *y*-intercepts of $3x - 5y = 11$.

We will use three basic techniques of graphing:

1. Find the intercepts and graph the line.
2. Make a table of values by substituting in several values of *x*, plot the points, and then draw the line.
3. Graph using the slope and y-intercept.

Example 2: Graphing by using intercepts.

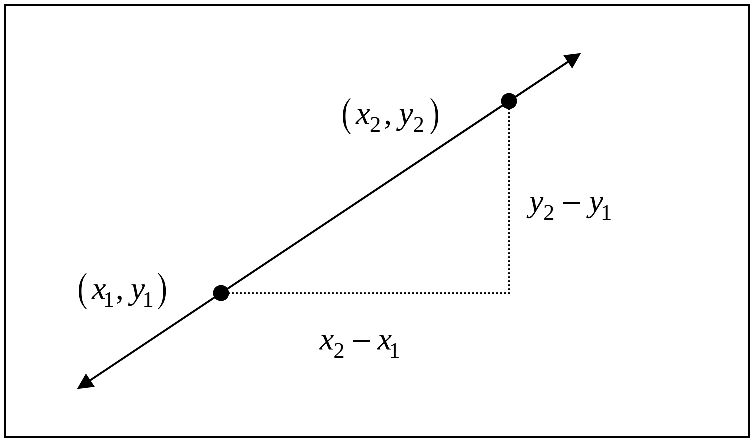
Find the *x*-intercept and *y*-intercept and graph: $2x - 4y = 6$



Slope

Definition: The **slope** of a line through two points is the ratio of the vertical change to the horizontal change. The slope of the line through the points (x_1, y_1) and (x_2, y_2) is

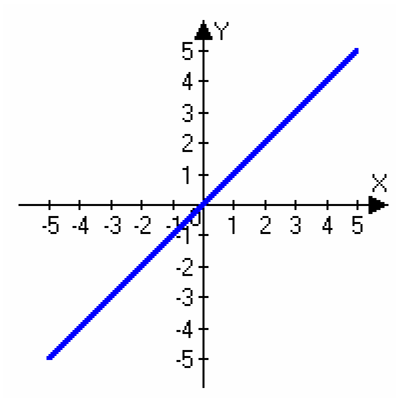
slope = $m = \dfrac{vertical\ change}{horizontal\ change} = \dfrac{\Delta y}{\Delta x} = \dfrac{y_2 - y_1}{x_2 - x_1}$



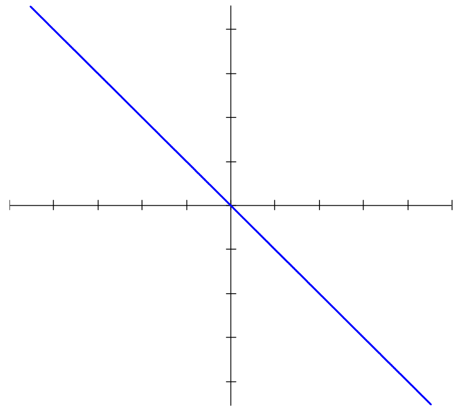
Example 3:
Find the **<u>slope</u>** of the line joining the points $(5, -3)$ and $(2, 3)$.

Facts about Slope:

1. The slope of a horizontal line is zero.
2. A line that rises going from left to right has **positive slope**.
3. A line that falls going from left to right has **negative slope**.
4. The slope of a vertical line does not exist. Why?



positive slope

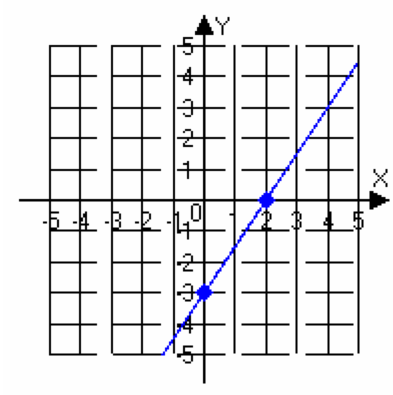


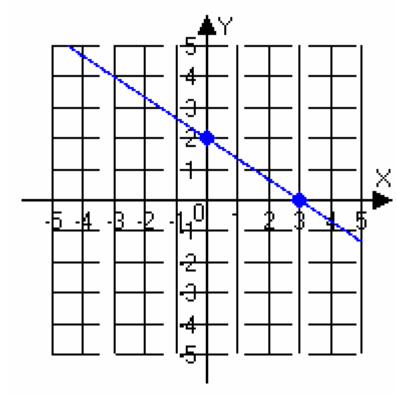
negative slope

Example 4: Find the **slope** of the line joining the points $(2,3)$ and $(-5,3)$

Example 5: Find the **slope** of the line joining the points $(3,-3)$ and $(3,-2)$.

Example 6: Finding the slope of a line given the graph.





Slope-Intercept Form of the Equation of a Line

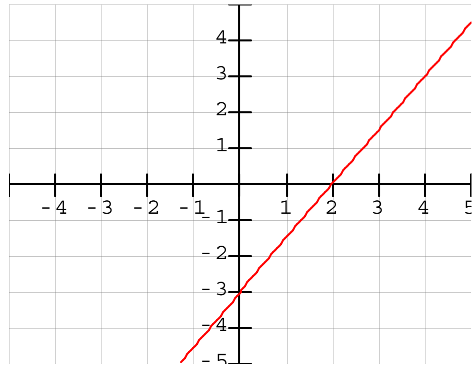
Definition: The **slope-intercept form** of the equation of a line is $y = mx + b$, where m is the slope and b is the *y*-intercept.

Example 7: Slope-intercept form.

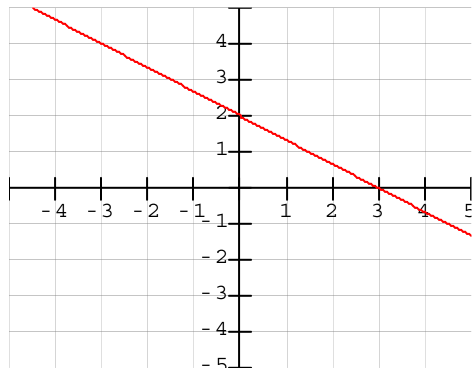
a) Determine the slope and *y*-intercept of $3y - 2x = 7$.

b) Give the slope-intercept equation for the line that passes through the point $(0, 4)$ with slope $m = 2/5$.

c) Find the slope-intercept equation for the line whose graph is given.



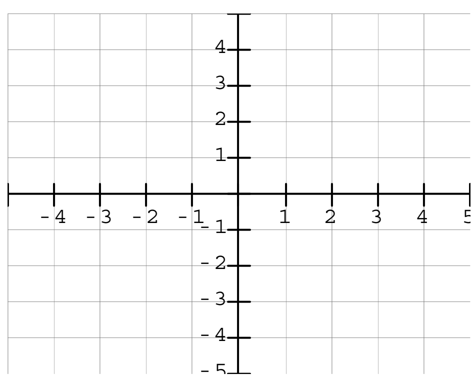
d) Find the slope-intercept equation for the line whose graph is given.



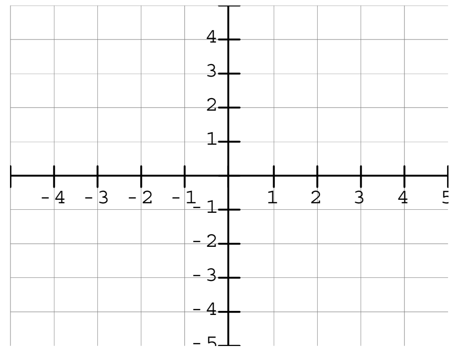
Example 8: Graphing using slope and y-intercept.

Graph the line from the given information.

a) Graph the line with slope $\frac{2}{5}$ and y-intercept $(0,-2)$.

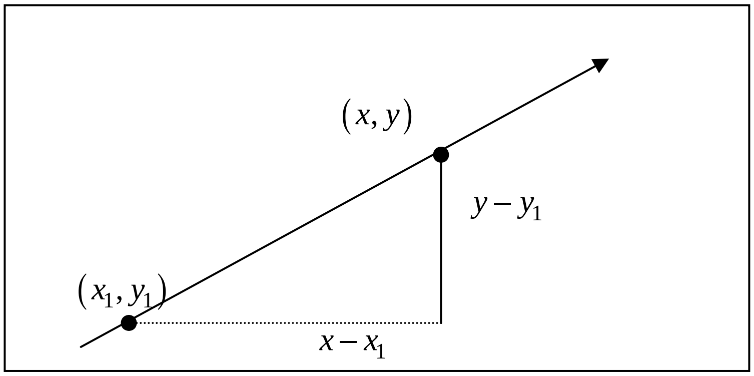


b) Determine the slope and *y*-intercept of $4y - 2x = 8$ and then graph.



Section 6 More Lines

Point-Slope Form of the Equation of a Line



The point-slope form is found by applying the slope formula to the graph above:

$$m = \frac{y - y_1}{x - x_1}$$

If we put the *y*'s on one side:

$$y - y_1 = m(x - x_1)$$

Example 1: **Finding Linear Functions**
Write a linear function that passes through the points (-2,5) and (4, -2).

Example 2: **Writing Equations of Lines**
Write an equation for the line that passes through the points (-2,5) and (-2, -2).

Example 3: **Writing Equations of Lines**
Write an equation for the line that passes through the points (-2,5) and (4,5).

Example 4: **Application: Finding Linear Functions**

The average cost of a home in Austin was $99,700 in 1990 and $186,400 in 2001. Write a linear function that gives the cost of a home in Austin as a function of x, the number of years since 1990. Use the function to predict the average cost of a home in Austin in 2010.

Parallel and Perpendicular Lines

Definitions
Two lines are **parallel** if they have the same slope: $m_1 = m_2$.
Two lines are perpendicular if the product of the slopes is -1: $m_1 \cdot m_2 = -1$. This means that the slopes are negative reciprocals.

Example 5: Using the point-slope formula
Write an equation of the line parallel to the line $-3x + 5y = 11$ and passing through $(-2, 3)$.

Example 6: Using the point-slope formula
Write an equation of the line perpendicular to the line $-3x + 5y = 11$ and passing through $(-2, 3)$.

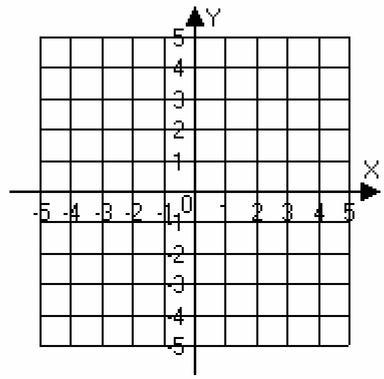
Special Cases: Horizontal and Vertical Lines

1. The graph of $x = a$ is a **vertical line** that passes through $(a, 0)$.
2. The graph of $y = b$ is a **horizontal line** that passes through $(0, b)$.

Example 7: **Graphing horizontal and vertical lines**

Graph the following lines.

a) $x = 3$

a) 

b) $y = -2$

b)