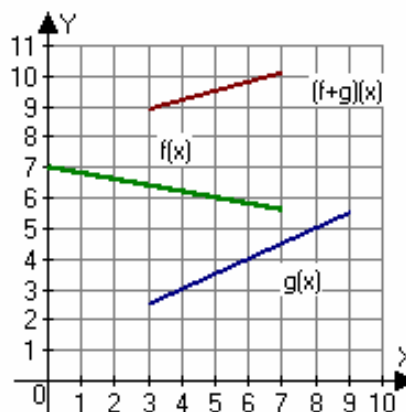
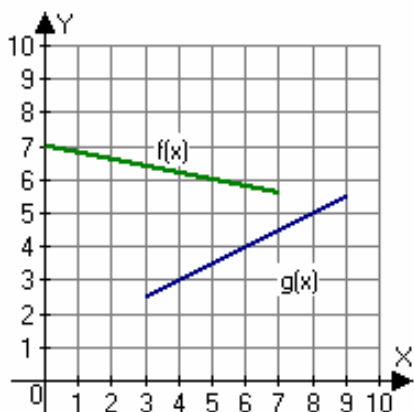


Section 7 The Algebra of Functions

Functions can be treated as objects upon which the usual operations of mathematics: addition, subtraction, multiplication, and division can be applied.

Two functions can be combined under certain conditions for values of x that are in both functions domains. For example, the graph below shows two functions. The domain of f is $0 \leq x \leq 7$ and the domain of g is $3 \leq x \leq 9$. The two functions can only be added where they are *both* defined. The domain of the sum function $(f + g)(x)$ is the intersection of the two domains: $3 \leq x \leq 7$. The function $(f + g)(x)$ is created by adding the output values corresponding to a given input value.

For example, $f(3) = \frac{13}{2}$ and $g(3) = \frac{5}{2}$, so $(f + g)(3) = f(3) + g(3) = \frac{13}{2} + \frac{5}{2} = \frac{18}{2} = 9$. In general, $(f + g)(x) = f(x) + g(x)$.



Suppose f and g are functions, then the following are also functions:

- The sum function $(f + g)(x) = f(x) + g(x)$
- The difference function $(f - g)(x) = f(x) - g(x)$
- The product function $(fg)(x) = f(x)g(x)$
- The quotient function $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$, $g(x) \neq 0$

Facts:

- The domain of $f \pm g, fg$ is the intersection of the domains of f and g .
- The domain of $\frac{f}{g}$ is the intersection of the domains of f and g except any value for which $g(x) = 0$ must also be excluded.

Example 1: Combining functions using numerical representations

x	0	1	2	3
$f(x)$	5	4	3	2

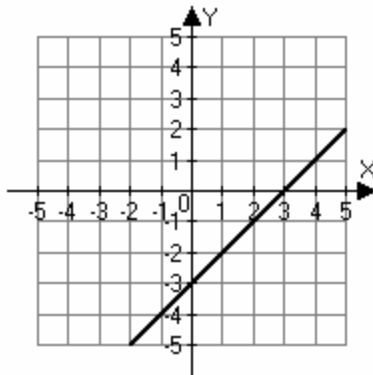
x	0	1	2	3
$g(x)$	1	2	0	3

- a) Find $(f + g)(2)$
- b) Find $(f - g)(1)$
- c) Find $\left(\frac{f}{g}\right)(1)$
- d) Find $\left(\frac{f}{g}\right)(2)$

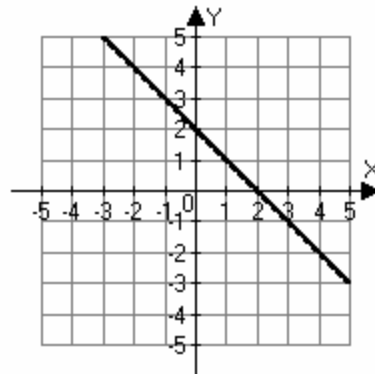
Example 2: Combining functions using graphical representations

Let f and g be defined as in the graphs below. Find the following, if possible, if not possible explain why:

$f(x)$



$g(x)$



- a) $(f + g)(0)$
- b) $(g - f)(0)$
- c) $(f - g)(2)$
- d) $(fg)(3)$
- e) $\left(\frac{f}{g}\right)(3)$

Example 3: Combining functions symbolically

Let $f(x) = x^2 + 1$ and $g(x) = x + 1$. Find the following functions and their domains.

a) $(f + g)(x)$

b) $(f - g)(x)$

c) $(fg)(x)$

d) $\left(\frac{f}{g}\right)(x)$

e) $\left(\frac{g}{f}\right)(x)$

Example 4: Combining functions symbolically

Let $f(x) = x^2 + 3x + 5$ and $g(x) = 2x - 15$. Find the following functions and their domains.

a) $(f + g)(x)$

b) $(f - g)(x)$

c) $(fg)(x)$

d) $\left(\frac{f}{g}\right)(x)$

Example 5: Combining functions symbolically

Let $f(x) = \frac{2x}{x+1}$ and $g(x) = 2x + 5$. Find $\left(\frac{f}{g}\right)(x)$ and its domain.