Linear

2.2 #2) $x^2 y' + 3xy = \frac{\sin x}{x}$ $\qquad x < 0$

$y' + \frac{3}{x} y = \frac{\sin x}{x^3}$ $\qquad \mu(x) = e^{3\int \frac{dx}{x}} = e^{3 \ln x} = x^3$

$(x^3 y)' = \sin x$

$x^3 y = \int \sin x \, dx$

$x^3 y = -\cos x + C$

$y = -\frac{\cos x}{x^3} + C x^{-3}$

#3) $y' + (\tan x) y = x \sin 2x$ $\qquad e^{\int \tan x \, dx} = e^{-\ln \cos x} = \sec x$

$(\sec x \, y)' = 2x \sin x \cos x \cdot \sec x$

$(\sec x) y = \int 2x \sin x \, dx$

$u = x \quad dv = \sin x \, dx$

$du = dx \quad v = -\cos x$

$= 2(-x\cos x + \int \cos x \, dx)$

$= 2(-x\cos x + \sin x + C)$

$(\sec x) y = -2x\cos x + 2\sin x + C$

$y = -2x\cos^2 x + 2\sin x \cos x + C\cos x$

or $y = -2x\cos^2 x + \sin 2x + C\cos x$

#4) $xy' + 2y = e^x$ $\quad , x > 0$

$y' + \frac{2}{x} y = \frac{e^x}{x}$ $\qquad e^{\int \frac{2}{x} dx} = e^{2 \ln x} = x^2$

$(x^2 y)' = x e^x$ $\qquad \int x e^x$ $\qquad u = x \quad du = dx$

$x^2 y = \int x e^x dx$ $\qquad dv = e^x \quad v = e^x$

$x^2 y = x e^x - e^x + C$ $\qquad \int u\,dv = uv - \int v\,du$

$y = \frac{e^x}{x} - \frac{e^x}{x^2} + \frac{C}{x^2}$ $\qquad = xe^x - \int e^x dx$

$= xe^x - e^x + C$

or

$y = x^{-1} e^x - x^{-2} e^x + C x^{-2}$

Linear

2.2 #6 $xy' + y = e^x \qquad y(1) = 1$

$$y' + \frac{1}{x} y = \frac{e^x}{x} \qquad p(x) = \frac{1}{x} \qquad \mu(x) = e^{\int \frac{dx}{x}} = e^{\ln x} = x$$

$$(yx)' = e^x$$
$$yx = e^x + C$$
$$y = x^{-1}e^x + Cx^{-1}$$
$$1 = e + C \Rightarrow C = 1 - e \qquad x >$$

$$y(x) = x^{-1}e^x + (1-e)x^{-1} \qquad x > 0$$

7) $y' + (\cot x)y = 2\csc x \qquad y\left(\frac{\pi}{2}\right) = 1$

$$p(x) = \cot x \qquad \mu(x) = e^{\int \cot x \, dx} = e^{\ln \sin x} = \sin x$$

$$(\sin x \; y)' = 2$$
$$\sin x \cdot y = 2x + C$$
$$y = \frac{2x}{\sin x} + \frac{C}{\sin x}$$

$$1 = \frac{\pi}{\sin \pi/2} + \frac{C}{\sin \pi/2} = \frac{\pi + C}{1}$$

$$1 = \pi + C$$

$$C = 1 - \pi$$

$$y = \frac{2x + 1 - \pi}{\sin x} \qquad 0 < x < \pi$$

Linear

2.2 #14) $y' - \left(\frac{1}{x}\right) y = x^{1/2}$

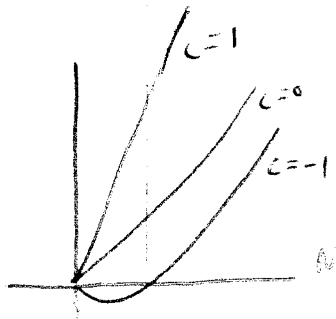
$\mu(x) = e^{-\ln x} = \frac{1}{x}$

$\mu(x) = \frac{1}{x}$

$\left(\frac{1}{x} y\right)' = x^{-1/2}$ $\quad x > 0$

$\frac{1}{x} y = \int x^{-1/2}\, dx = 2 x^{1/2} + c$

$y(x) = 2 x^{3/2} + cx$



Notice that curves all go through origin. Happens where $\frac{1}{x}$ is undefined so conditions of existence and uniqueness theorem break down.

#15) $y' + \left(\frac{1}{x}\right) y = \frac{\cos x}{x}$ $\qquad \mu(x) = e^{\ln x} = x$

$(xy)' = \cos x$

$xy = \int \cos x$

$xy = \sin x + c$

$y = \frac{\sin x}{x} + c x^{-1}$